**A113 – Mathematics**

**Worksheet for Problem 05: Pushcart**

**Application of straight line equation**

1. **National Education Message:**

*“No one owes Singapore a living”* – we find our own way to survive and prosper, turning challenge into opportunity.

While Singapore’s average income has risen significantly through the years, there are still families who face difficulties coping with their daily needs.

Discuss the above National Education Message.

a) What do you think is Service-Learning?

b) List a few ways on how you think service learning can help the needy families in Singapore.

1. Look at Table 1 in the problem statement and observe the trend from past charity drives.
2. Open the attached MS Excel file, **Pushcart.xlsx.** Fill in the relevant columns in the worksheet titled “Demand” using the data from Table 1 in the problem statement. Plot the graph of *Q* versus *P*.



1. Describe how the quantity of mugs sold, *Q*, changes with the unit price of the mug, *P*.

The lower price it is, the more quantity sold.

1. Suggest one possible equation relating *Q* and *P* for the plotted graph. (Hint: Make use of what you have learnt in P04 – Getting It Straight)

**Q = -100P+1450**

**Relationship between proceeds, revenue and cost**

1. Can we assume that the amount of money collected from the charity drive, also known as revenue, equals to the amount of proceeds? Why?

No. This is because proceeds is minus costs.

1. Discuss if a higher price necessarily means higher proceeds. Why?

No. Usually there us a maximum price a consumer is willing to pay.

**Interpreting graph**

Let us now try to determine how the proceedswill vary with different values of *P*.

1. Complete the worksheet titled “Proceeds” (in Pushcart.xlsx) using the relevant data. Plot a graph of proceeds versus price in the worksheet.

Referring to the graph you have plotted,

1. What do you notice about the shape of the graph?

It is a parabola that opens downwards. Quadratic graph.

1. What does the portion of the graph above the *P*-axis (price) represent? What about the portion below the *P*-axis?

It represents the price. They made a loss.

1. State the value of the proceeds when *P* = 0 and explain what does the value represent.

-700. Rental cart costs.

1. At what point(s) on the graph would there be zero proceeds and zero loss (i.e. a break-even point) for the charity drive? Hence, what is/are the values of  that will allow you to achieve break-even?

(0.50,0) and (14, 0)

$0.50 and $14.00

**Maximising proceeds using graphical method**

1. What value of  would allow us to achieve the maximum proceeds? What would the proceeds be then?

7 and 7.50. It would be your maximum point.

1. If the target for this charity drive is to achieve at least a proceeds of $1,100, use the graph plotted in Q5 to determine the minimum price you should sell the mugs in order to achieve this target.

$1.50

1. Suppose the target is now raised to $1,200, how would your answer in Q7 be affected?

It would not be affected.

Using graphs to determine the value(s) of price that will result in specific values of proceeds is relatively easy to do and provide a good way to visualise how the proceeds changes with.

**Forming the graph equation**

Besides the graphical method, there are two algebraic methods to determine maximum proceeds: (i) **quadratic formula** and (ii) **completing the square**. For both algebraic methods, the equation describing the graph plotted in Q5 will be needed.

1. Determine the equation by considering the following:
2. Suppose the monthly revenue can be represented by *R*, express *R* in terms of *Q* and *P*.

R = P\*Q

1. Using the equation your team suggested in Q2(c) and Q9(a), express *R* in terms of *P*.

R = -100P2 +1450P

1. Hence, if the proceeds is represented by *T*, how would you express *T* in terms of *P*?

T = R-700

The equation derived is a quadratic equation. In general, a **quadratic equation** can be written in the form of, where *a*, *b* and *c* are constants.

**Maximising proceeds using quadratic formula**

1. Let us find out more about the quadratic formula in the attached MS Word file, **Quadratic Formula.doc**:



1. Using the quadratic formula , find the values of *P* that will result in zero proceeds (i.e. *T* = 0), correct to 2 decimal places. Verify that the values of *P* determined by the methods in Q5(d) and Q11 are the same.

0.5 and 14. The values are the same.

1. From Q11, are you able to determine the value of  that would allow us to achieve the maximum proceeds? (Hint: The quadratic curve is symmetrical about the lowest or highest point of the curve)

7 and 7.50

What would the proceeds be then? Verify your answers with those that you have obtained in Q6.

Yes, it is the same.

**Maximising proceeds using completing the square method**

1. First, let us see how we can use the completing the square method based on a simple example in the attached MS Word file, **CTS.doc**:



1. Let us apply what we have learnt from Q13 to determine the maximum proceeds. The quadratic equation  can be expressed as follows:

,

where ,  and are constants.

1. Using appropriate values for *a*, *b* and *c*, express the equation you have derived in Q9(c) in a form that is similar to the above expression.
2. Based on the expression which you have derived from part (a), determine the maximum proceeds *T*, and the associated price, *P*. Verify that the maximum proceeds determined by the method in Q6 and this question is the same.
3. Find the values of *P* that will result in zero proceeds (i.e. *T* = 0), correct to 2 decimal places.

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| ***Practice Questions***  *(It is essential to complete these practice questions so that you can understand the concepts of this lesson better and be more confident and competent in handling related questions.)*  Application of straight line equation  [You may want to watch the following [video](https://docs.google.com/file/d/0BxYqWZfRUFnoUGExc2tzZUcwTWM/edit?usp=sharing) which would help to recap some of the key concepts learnt before attempting the following question.]  Given that a straight line passes through the points (1 , 3) and (2 , 4), form the equation for this straight line in the form of  The gradient of a straight line, which passes through the point (-1, -5), is 4. What is the equation of this straight line?  Forming quadratic equation  The dimensions of a rectangular box are given as follow:  Length: *k* units Breadth: (*k +* 3) units Height: 15 units  Form an equation in terms of *k* to represent the volume of the box.  The perimeter of a right-angled triangular field as shown below is 50 units. Given that the dimensions for two sides of the field are () units and 22 units, form an equation in terms of *x* to represent the area of the field.    22  Field  Quadratic formula  Apply the quadratic formula to find the value(s) of *x* for the following quadratic equations:          Completing the square  [You may want to watch the following [video](https://docs.google.com/file/d/0BxYqWZfRUFnoVDZoZTVxRzhaLXc/edit?usp=sharing) which would help to recap some of the key concepts learnt before attempting the following question.]  Express the following quadratic equations in the form of a ‘completed square’.    *y* = ( )2    *z* = ( )2    *w* = ( )2    *p* = ( )2  From the equation  Find the value(s) of *y* when *x* = 0.    Find the value(s) of *x* when *y* = 0.    What is the minimum value of *y* for this graph?  State the equation for the line of symmetry.  The figure below shows a quadratic curve,.  ‘-[   1. What is the value of *k* given that Q is the highest point on the graph?   *k* =   1. Given that *c* = -12, find the values of *a* and *b*.   *a* =  *b* =   1. Hence calculate the value of *m*.   *m* =  For each of the following equations, determine the *y*-intercept, *x*-intercepts and maximum/minimum point of the quadratic curve.      *y*-intercept =  *x*-intercepts =  Maximum/Minimum point =      *y*-intercept =  *x*-intercepts =  Maximum/Minimum point =  10. Given the following quadratic equation  .   1. Determine the coordinates of the points where the quadratic curve intersects the *x*-axis.   ( , ) and ( , )   1. Using the Completing the Square method, determine the coordinates of the minimum point.   ( , )   1. What is the relationship between the *x*-coordinates of your answers in part a and b?   11. An object is launched at 19.6 meters per second (m/s) from a 58.8-meter tall  platform. The equation for the object's height *s* at time *t* seconds after launch above  the ground is , where *s* is in meters.  Calculate the maximum height *s* and corresponding time *t* the object can reach.    *s* =    *t* = |

**Putting it together**

15. Use the ideas explored in the sections above, respond to the day’s challenge

for your team.

**Exploring further**

16. Given the quadratic equation,  how would you derive the quadratic formula ?

17. Suppose the quadratic equation representing the graph of proceeds

against price is given as follows:



(i) Find the value of .

(ii) Hence describe the nature of roots of the proceeds function.

18. Suppose the quadratic equation representing the graph of proceeds against

price is given as follows:



1. Applying the methods that you have learnt so far, can the values of *P* be found for the above equation when *T* = 0? Why?
2. Determine the maximum proceeds *T*, and the associated price, for the above equation.